Moduli Stabilization
in the Heterotic/IIB Discretuum

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Abstract

We consider supersymmetric compactifications of type IIB and weakly coupled heterotic string-theory in the presence of G resp. H-flux and various non-perturbative effects. We point out that non-perturbative effects change the Hodge structure of the allowed fluxes in type IIB significantly. In the heterotic case it is known that, in contrast to the potential read off from dimensional reduction, the effective four-dimensional description demands for consistency a non-vanishing $H^{2,1}$ component, once a non-trivial $H^{3,0}$ component balances the gaugino condensate. The $H^{2,1}$ causes classically (but not when non-perturbative effects are included) a non-Kähler compactification geometry whose moduli space is, however, poorly understood. We show that the occurrence of $H^{2,1}$ could be avoided with world-sheet instantons by using a KKLT-like two-step procedure for moduli stabilization. Moreover, heterotic moduli stabilization under the inclusion of one-loop corrections to the gauge kinetic function led to negative gauge couplings and a corresponding strong coupling transition. This problem disappears, as well, when world-sheet instantons are included. They stabilize moreover the Kähler modulus without the need for a non-Kähler geometry with non-trivial dJ.

Keywords: Moduli Stabilization, Fluxes, Type IIB/Heterotic String-Theory

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1 Introduction and Summary

In this paper we will investigate the problem of moduli stabilization in type IIB and heterotic string supersymmetric flux compactifications with additional contributions to the effective superpotentials, most notably gaugino condensation or also wrapped Euclidean D3-branes. We will see that the discrete landscape considerably differs, notably in the Hodge types of the allowed flux, from those groundstates obtained from a pure 3-form flux superpotential.

On the type IIB side the analysis is to very large extent motivated by the KKLT scenario [1] where the 3-form flux superpotential is augmented by additional terms which depend on a size modulus of the compact internal space X. On the heterotic side we will study the combined effect of an effective superpotential which contains the NS 3-form flux H, the gaugino condensate, world-sheet instanton effects and possibly terms that describe the deviation from X being Kähler. Both the heterotic and the type IIB case exhibit several similarities and analogies, which should be explainable from some underlying string-string duality symmetry. We will exhibit these analogies and discuss them in the context of the effective actions, but we will not give a serious attempt to trace them back to some string-string duality transformations (which should nevertheless be possible in some explicit orientifold/heterotic dual pairs). In both cases the result of the discussion will be rather similar: the inclusion of the additional effects in the superpotential besides the 3-form fluxes has the effect that generic supersymmetric groundstates are described in the type IIB case by fluxes which are not anymore ISD (imaginary self-dual) with only $G^{2,1}$ and $G^{0,3}$ components but rather will include all IASD (imaginary anti self-dual) types as well, respectively in heterotic compactifications there will generically be H-fluxes of type (2,1) and (1,2) besides the 'usual' (3,0) H-flux. We show the necessity of the more general Hodge type by arguing that $H^{2,1} = 0$ is generically impossible to impose consistently. Only in a kind of two step procedure (which constitutes an additional assumption), applied in the original work of KKLT, where one first fixes the complex structure moduli, and then solves the supersymmetry conditions for the remaining fields, the original Hodge structure for G- resp. H-flux can be preserved.

Superpotentials and moduli stabilization in heterotic compactifications

The procedure used in the heterotic string theory bears a number of interesting similarities and important differences in comparison with the type IIB situation. Firstly the complex structure moduli z_i are now fixed by enforcing a proportionality between $H^{3,0}$ and Ω; here the first is being assumed to get contributions just from the dB sector (as is the case in the standard embedding). The quantization [2] of $H = dB$ leads

then to a corresponding fixing of the periods. The needed proportionality stems from a balancing between the gaugino condensate in the hidden sector and the $H^{3,0} + c.c.$ flux. This, and some details of this situation that are recalled below in subsect. 3.1, comes from the corresponding complete square in the heterotic string Lagrangian. While this mechanism alone, realising a no scale scenario, will break supersymmetry and leave the T-modulus unfixed with a still vanishing (tree-level) potential, it can be promoted to a T-stabilizing supersymmetric AdS model by including size-dependent effects like one-loop gauge coupling corrections.

It was shown [3] that in heterotic compactifications it is not possible to turn on consistently (in a supersymmetric way, say in the z_i and S sectors) the flux component $H^{3,0} + c.c.$ *alone*. A component $H^{2,1} + c.c.$ will be induced automatically (even if it is only a small quantum effect). Having such a component in the heterotic string implies [4] *classically* that the underlying compactification manifold cannot be Kähler any longer if supersymmetry is to be preserved (some further details from more recent investigations on this set-up will be recalled below, too). But in that case it is not quite clear what the appropriate moduli replacing or generalising the complex structure moduli and the Kähler moduli will be; therefore one would have, taking this seriously, a somewhat inconsistent starting point when using superpotentials for the ordinary moduli. Hence one would prefer to avoid the occurrence of the dJ component.

A strong argument in favor of having a non-trivial $dJ \neq 0$ has been its stabilization effect on the overall radial modulus T [4], [5], [6]. In this work we will show in section 4 that by adding the size-dependent world-sheet instanton effect

$$W_{WSI} = Be^{-bT} \tag{1.1}$$

to build the combined superpotential

$$W = W_H[z_i] + W_{WSI}[T] + W_{GC}[S] , \tag{1.2}$$

one can stabilize all moduli supersymmetrically, including the radial modulus T, without the need for a non-trivial dJ. In view of the unknown moduli space of non-Kähler spaces this is an interesting result which allows to carry out the moduli stabilization program rigorously within the currently known mathematical framework.

We first discuss this in the scenario where the complex structure moduli do not acquire masses above the mass scale for the stabilized T-modulus and therefore do not decouple from its dynamics. Here we point out that when solving the supersymmetry conditions for all moduli with vanishing $H^{2,1}$ one generically overconstrains the H-field. More precisely, when one wants to allow for a potential solubility of the constraints even in principle, it

is crucial to take into account the non-trivial dependence of the one-loop determinant B on the complex structure moduli z_i

$$B = \frac{\text{Pfaff}\,\bar{\partial}_{V(-1)}|_C}{(\det\bar{\partial}_{\mathcal{O}(-1)}|_C)^2} \ . \tag{1.3}$$

(here a constant is suppressed and actually for reasons of well-definedness the exponential T factor should be included, cf. [7]). C is a contributing genus zero curve which is assumed to be isolated so that its normal bundle[4] is $N = \mathcal{O}(-1) \oplus \mathcal{O}(-1)$ (we will not consider the dependence of B on vector bundle moduli in this paper). That is, it is important to note that

$$\partial_{z_i} B \neq 0 \ , \tag{1.4}$$

which violates the seemingly decoupled structure of the three contributions in (1.2) with respect to the z_i, T and S dependence. But, since a suitable adjustment of H can nevertheless not be ensured generically, we also discuss the two-step procedure as an alternative.

Comparison with moduli stabilization in type IIB

Recall that in [10] effective supergravity descriptions for type IIB with $D3$ and/or $D7$ branes along the lines of [1] and [11] resp. a heterotic theory with flux and gaugino condensation were considered in parallel. On the type IIB side this amounts to the consideration of the superpotential

$$W = A + B\tau + Ce^{-aT} \qquad \text{with } A = \int F \wedge \Omega \ , \ \ B = -\int H \wedge \Omega \tag{1.5}$$

or, when the dependence of A and B on the complex structure moduli z_i is considered, to the investigation of $W = W^{\tau}_{eff}[\tau] + Ce^{-aT}$ after the z_i have been integrated out.

On the heterotic side, after the replacement $\tau \to T$ and $T \to S$ the superpotential is

$$W = A + BT + Ce^{-aS} \qquad \text{with } A = W_H = \int H \wedge \Omega \ , \ \ B = -\frac{i}{2}\int dJ \wedge \Omega. \tag{1.6}$$

We expect that inclusion of the dJ completion of H makes for a full analogy to the type IIB situation in many respects. In the following we do not invoke the term $\frac{i}{2}\int \Omega \wedge dJ$ so that we have $B = 0$. The omission of this term, which keeps H purely real, will be the reason for the impossibility to stabilize a complex structure modulus z exponentially near to a conifold vacuum $z = 0$, cf. the remarks after (3.44). Further one has the problem of weak coupling stabilization of the heterotic dilaton. To draw the parallel to the type IIB

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[4]The denominator of (1.3) is $\det \partial_{y_i y_j} W_2$ with the superpotential $W_2(C = \partial D) = \int_D \Omega$ (up to additive constants) and y_i local coordinates of N. W_2 is stationary for holomorphic C [8], cf. [9].

procedure, when speaking next about complex structure moduli z_i, we restrict (having $B = 0$) to the Kähler case as otherwise a rationale for grouping the moduli in 'Kähler moduli' and 'complex structure moduli' (both may be in some generalized sense) is not well understood; pragmatically we consider (here for the case without the one-loop corrections) with [10] a decomposition of W where one has $W = W_{eff}^T[T] + W_{GC}$, i.e.

$$W_{eff}^T[T] = W_H + Be^{-bT} \tag{1.7}$$

Using this 'substitution'-dictionary $T \leftrightarrow S$ between type IIB and the heterotic theory one finds that a number of problems of the heterotic theory can be addressed successfully, partly when the two-step procedure is employed.

Firstly, the pertinent problem of stabilization of the heterotic dilaton at weak coupling (large S_R) mirrors the analogous problem of the consistency of the SUGRA approximation in the type IIB investigation: there too it was found that at the minimum (or rather: stationary point) of the superpotential $W = W_{flux} + W_{non-pert}$ (with $W_{non-pert} = Ce^{-aT}$ or Ce^{-aS} in type IIB and the heterotic theory, respectively) the two contributions have to balance each other approximately. This leads to the analogous problem as one encounters in the heterotic dilaton stabilization: for consistency of the analysis one needs large T_R (resp. S_R) but the flux is integral and the period generically of order 1. It is then the degree of freedom which stems from having many 3-cycles which comes to the rescue in the type IIB case thereby making possible a exponentially small W_{flux}. There are important differences to this in the heterotic case (cf. discussion in subsect. 3.6). It is also interesting to note that if one would try to make W_{flux} small by having just one flux on a conifold cycle and tries to make the corresponding z-period small as in [12] one encounters the difficulty that the heterotic 3-form is *real* and therefore the near-conifold vacuum (z exponentially close to zero) can not be stabilized.

Secondly adopting the two-step procedure of KKLT in the heterotic case by fixing the z_i first by using just the W_{flux} alone, one can avoid the occurrence of a $H^{2,1}$ component. In the paper we will write down the $D_iW = 0$ conditions first in the full form and then point to the emerging $H^{2,1}$ resp. its avoidance indicated here. In this case we find that the remaining dilaton and radial moduli, S and T, can both be stabilized reliably without the need for a non-trivial dJ.

Here we spent some time to show that the greater flexibility in the heterotic string of having not just $H = dB$ will not be enough to avoid $H^{2,1}$ independently of the use of the two-step procedure. One might have guessed that this could be possible as the relevant condition that $H = dB - (CS_{YM} - CS_L)$ has type $3, 0 + c.c.$ does not lead immediately to a second set of conditions on the periods (as in [2]) as the CS sector is not quantized

(the usual argument for z_i stabilization relies on dB, and then therefore also a multiple of $\Omega + c.c.$, being integrally quantized [2]). But this sector is difficult to control (as it comes naturally with its own supersymmetry conditions, cf. below) if A_{YM} is not just flat (cf. [13]) in which case CS_{YM} is again quantized.

Avoiding the strong coupling transition problem

Furthermore, and independently of the $H^{2,1}$ issue, we will try to solve the strong coupling transition puzzle which arose in [13] where vacua based on $H^{3,0}$ flux and gaugino condensation contributions were considered. It was found there that one-loop corrections $f_{obs/hid} = S \mp \beta T$ to the gauge couplings led to negative gauge couplings

$$\mathrm{Re} f_{obs} < 0 \ . \tag{1.8}$$

This would have implied a not well understood strong coupling transition. We will show that once world-sheet instantons are additionally taken into account this problem disappears. Likewise the unorthodox choice $\mathrm{Re} f_{obs} < \mathrm{Re} f_{hid}$, which was found to be necessary in [13], will no longer be needed.

We finally prove the stability of the vacuum along the lines of [10] by checking the stability criterion of [10] actually for the relevant $W_{eff}^T[T]$.

The paper is structured in the following way. In the next section we will discuss the supersymmetric ground states in type IIB with 3-form fluxes as well as with additional contributions to the superpotential and point to the occurrence of IASD flux components even in the supersymmetric case. Then we will introduce the heterotic superpotential and exploit several analogies with the type IIB superpotential. We will discuss the possibility of having just an $H^{3,0}+c.c.$-flux balancing the gaugino condensate and show that it is only possible to avoid the occurrence of a further $H^{2,1} + c.c.$-flux by implementing a KKLT-like two-step procedure also into the heterotic compactifications. Note that $dJ \neq 0$ is no longer simply enforced by $H^{2,1} \neq 0$ (as in the classical case $D_i W_{flux} = 0$) when non-perturbative effects are included which now (instead of the previous dJ) balance an $H^{2,1}$. One positive feature of having a dJ component was that it would fix the size of the overall radial modulus we proceed to discuss an alternative mechanism to achieve this. We will investigate the inclusion of a non-perturbative size-fixing heterotic superpotential from world-sheet instantons. In this framework we can solve furthermore the strong coupling transition problem which was present in T-modulus stabilization from including one-loop corrections to the gauge kinetic functions. We finally check the stability of the vacuum along the lines of [10].

2 Moduli Stabilization in Type IIB with 3-Form Fluxes and Non-Perturbative Corrections

Let us now consider the type IIB case and discuss which are the conditions on the 3-form flux G for supersymmetric vacua taking into account Kähler moduli dependent corrections to the flux superpotential, cf. [1], [10]. Since there is a striking analogy between the type IIB and the heterotic moduli stabilization procedures, we will be particularly interested in the question under which conditions the $G^{1,2}$ flux component arises; this will help us to better understand later the occurrence of the heterotic $H^{2,1} + c.c.$ component. Specifically we are considering the following type IIB superpotential

$$W = W_G + W_{D3I} + \left[W_S \right] = \int G \wedge \Omega + C(z_i)\, e^{-aT} \quad \left[+ C'(z_i)\, e^{-bS} \right] \tag{2.1}$$

Here $S = s + i\sigma_S$ denotes $-i\tau$ with τ the type IIB dilaton so that $G = F - \tau H = F - iSH$. T denotes[5], by abuse of language, the four-cycle volume of the $D3$ instanton. It is closely related to the proper Kähler modulus measuring a two-cycle volume. This difference will however not being important here.

The first term W_G is the standard type IIB flux superpotential

$$W_G = A + B\tau \qquad \text{with} \;\; A = \int F \wedge \Omega \;, \;\; B = -\int H \wedge \Omega \tag{2.2}$$

The supersymmetric vacua which follow from the flux superponetial are obtained if the flux is ISD and and of the form $G^{2,1}$ [12] (more discussion on flux vacua can be found e.g. [14]).

The second term W_{D3I} is the correction due to Euclidean D3-branes wrapped around 4-cycles in the X. For them to contribute to the superpotential the four-fold used for F-theory compactification has to admit divisors of arithmetic genus one, which project to 4-cycles in the base X. Alternatively, W_{D3I} can originate from non-perturbative gaugino condensation in some hidden, asymptotically free gauge group. The difference between wrapped Euclidean D3-branes and gaugino condensation will manifest itself in the constant a in the exponent of W_{D3I}. Note that the prefactor $C(z_i)$ is in general a complex structure moduli dependent function. In order to be fully general, and also in analogy to the heterotic case, we have included into W a third term, W_S, being an exponential in the type IIB dilaton $\tau = iS$, and being again equipped with a complex structure moduli dependent function $C'(z_i)$. This term could be motivated by the action

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[5]often called ρ in the literature; for the comparison with the heterotic string we switch the notation, cf. also [10]

of the D(-1)-brane instanton [15]. (One might also consider, here and in the heterotic theory, the inclusion of NS 5-brane instantons wrapping X.)

From the Kähler potential (assumed to have the standard tree level form)

$$K = K(z_i, \bar{z}_i) - 3\log(T + \bar{T}) - \log(S + \bar{S}) \tag{2.3}$$

one finds that demanding unbroken supersymmetry in the complex structure moduli z_i sector yields the conditions

$$D_i W = \int G \wedge \chi_i + \partial_i C \, e^{-aT} + K_i \, W_{D3I} \quad \left[+ \partial_i C' \, e^{-bS} + K_i \, C' \, e^{-bS} \right] = 0 \tag{2.4}$$

(here $D_i\Omega = \chi_i$, with the χ_i being a cohomology basis for (2,1)-forms). It follows that, with $W_{D3I} + W_\tau$ included, one needs $G^{1,2} \neq 0$ for unbroken supersymmetry.[6]

Similarly unbroken supersymmetry in the Kähler modulus sector yields

$$D_T W = -aC \, e^{-aT} - \frac{3}{2t} W = 0 \tag{2.5}$$

Therefore one needs $G^{0,3} \neq 0$ for unbroken supersymmetry. This Hodge component was implicitly already present in [1] when a non-zero value W_0 of W_G was discussed. It goes beyond the supersymmetric flux components $G^{2,1}$ of [12] when discussing just $W = W_G$ but is still ISD. But as we describe here the other two IASD components $G^{1,2}$ and $G^{3,0}$ will be present as well.

Finally the dilaton sector shows the need of a $G^{3,0} \neq 0$ for unbroken supersymmetry

$$D_S W = -\frac{1}{2s} \int \overline{G} \wedge \Omega - \frac{1}{2s} W_{D3I} \quad \left[-bC' \, e^{-bS} - \frac{1}{2s} C' \, e^{-bS} \right] = 0 \tag{2.6}$$

In conclusion, whereas supersymmetric vacua from a pure flux superpotential are obtained from ISD 3-fluxes $G^{2,1}$, the T- (and possibly S-) dependent corrections to W imply that supersymmetric groundstates correspond to more general ISD plus IASD fluxes[7] where all possible Hodge types are turned on. (Specific type IIB orientifolds of this type will be constructed in [16].) The supersymmetric minima obtained in this way are generically anti-de Sitter since the groundstate has the property $W \neq 0$.

One may ask whether this result is also important for the statistical counting of supersymmetric flux vacua [17]. So far in the literature the count for supersymmetric flux

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[6]Note that it seems not possible in general to set all $G^{1,2}$-fluxes to zero, which sets up a system of $n := h^{2,1}$ equations for n moduli fields, and to simultaneously set up a cancellation among the other terms in (2.4) since this requires to solve another set of n independent equations for the z_i.

[7]Neglecting W_S one gets $b_i^{1,2} = -K_i \bar{\alpha} - \partial_i C e^{-aT}/\frac{vol}{i}$ (adopting partially the later notation (3.11) here for the *complex G*), or $b_i^{1,2} = -K_i \bar{\alpha}$ (neglecting $\partial_i C$) relating the IASD components $G^{1,2}$ and $G^{3,0}$.

vacua was done under the assumption that the 3-flux is ISD. Including the corrections of the type discussed above will (as long as these are suitably small) amount to tiny shifts of the critical points.

Remarks on the use of the decoupling procedure

Let us discuss these results in the light of the two-step procedure applied in [1] for obtaining supersymmetric groundstates from the superpotential (2.1). In the first step groundstates of the pure 3-form superpotential W_G were found for which the 3-form flux is purely ISD. More precisely, as described in [12], the conditions $D_i W_G = 0$ are solved by considering only fluxes with $G^{1,2} = 0$ (the Hodge-components $G^{0,3}$ and $G^{3,0}$ are similarly set to zero if the conditions $W = 0$ and $D_\tau W = 0$ are imposed). This in general fixes all complex structure moduli.

In the second steps one plugs in the values for the fixed complex structure moduli into W_G (and also into $C(z_i)$) and then solves the supersymmetry conditions for the remaining Kähler modulus T. As now a non-zero value W_0 of W_G is employed one has gone already beyond a pure $G^{2,1}$ and has turned on a $G^{0,3}$, as already done in [1].

The work of [1] is based on this set-up and then crucially assumes that the z_i are much heavier than T such that they decouple and one merely remains with the stabilization problem for T. However without the assumption of the z_i being integrated out, i.e. without assuming the KKLT two-step stabilization procedure, the $D_i W = 0$ condition picks up a further contribution such as $K_i W_{D3I}$, which actually enforces a non-trivial $G^{1,2}$ component.

So the above analysis shows that this two step procedure is only of limited justification (similar conclusion were drawn in [10]). The generic situation is more appropriately captured by solving all supersymmetry conditions at the same time. Then supersymmetric 3-form flux is not only ISD, but all Hodge types appear.

The advocated two step procedure of [1] is only justified if the T- (or S-) dependent corrections are indeed suitably small, such that the flux is almost of Hodge type $(2, 1)$, and the complex structure moduli are fixed to values which are very close to those of a pure 3-form flux superpotential. Here it will be helpful to have many 3-cycles making possible to have an exponentially small W_G.

Then let us call W_0 the value of W_G for these moduli. This is now only a constant, generically non-vanishing, which just can occur in $K_S W$- or $K_T W$-parts of covariant derivatives of the full W. So, starting from $W = W_0 + Ce^{-aT}$ with a real and $K^{(T)} = -3\log(T + \bar{T})$, one finds after the cancellation of the $-3|W|^2$ part for the potential (with

$T = t + i\sigma_T$)

$$V = \frac{1}{8t^3}\left(\frac{1}{3}a\left(a + \frac{3}{t}\right)\left(2t|C|e^{-at}\right)^2 + 2tae^{-at}2\mathrm{Re}(C\bar{W}_0 e^{-aT})\right) \tag{2.7}$$

When adopting the often imposed condition $C, W_0 \in \mathbf{R}$ the second term in the brackets becomes $aW_0 2tCe^{-at}2\cos a\sigma_T$. This gives finally in the sector without axion-component

$$V|_{\sigma_T=0} = \frac{aCe^{-at}}{2t^2}\left(\left(\frac{at}{3} + 1\right)Ce^{-at} + W_0\right) \tag{2.8}$$

When employing this 'integrating out' procedure[8] a number of points should be addressed:

- In principle one has to make sure that the shift in z, caused by fixing it just by $W = W_G$ instead of using the full $W = W_G + W_{D3I}$, is appropriately small (for remarks on this cf. [18]).

- It must be checked that the stabilized z_i are more heavy than the stabilized T modulus (in principle this should be checked for the moduli values stabilized from the full superpotential, cf. the first point, not just for (2.8)).

- The stability of the stationary point has to be checked; this is a non-trivial point and not always a minimum is found [10].

3 Heterotic Moduli Stabilization with 3-Form Fluxes and Non-Perturbative Corrections

We will first recall the argument presented in [3] for a necessary emergence of the $H^{2,1}$ component and connect then to the stabilization of the dilaton also discussed in [13].

3.1 3-Form Fluxes in the Heterotic String

An important mechanism to break supersymmetry while still maintaining a vanishing cosmological constant at tree level stems from a complete square in the heterotic string Lagrangian suggesting a cancellation between the gaugino condensate and an H-flux [19]

$$\int d^{10}x\sqrt{-g}\left(H_{mnp} - \alpha'\mathrm{tr}\,\bar{\chi}\Gamma_{mnp}\chi\right)^2. \tag{3.1}$$

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[8]All of this concerns just the part of the argument before a potential 'up-lift' (by $\overline{D3}$-branes, say).

The gauginos condense non-perturbatively such that $\operatorname{tr}\bar{\chi}\Gamma_{mnp}\chi$ acquires a vacuum expectation value (vev)

$$\langle\,\operatorname{tr}\bar{\chi}\Gamma_{mnp}\chi\,\rangle = \Lambda^3\,\Omega_{mnp} + c.c. \,, \tag{3.2}$$

where Ω_{mnp} is the holomorphic $(3,0)$ form of the internal Calabi-Yau manifold and $\Lambda^3 = \langle\,\operatorname{tr}\bar{\lambda}_D\frac{1}{2}(1-\gamma_5)\lambda_D\,\rangle = \langle\operatorname{tr}\lambda\lambda\rangle$ the vev of the four-dimensional condensate. One gets Minkowski vacua of vanishing tree-level potential for the flux choice $H = H^{3,0} + c.c.$ with

$$H = \alpha'\,\Lambda^3\,\Omega + c.c. \tag{3.3}$$

The other Hodge component $H^{2,1} + c.c.$ can be turned on supersymmetrically only if the underlying compactification geometry possesses the balancing property of being non-Kähler [4], [20], [6] (cf. also [21], [22]) which allows for a nontrivial $\partial J \neq 0$ and thus

$$H^{2,1} + c.c. = \frac{i}{2}\partial J + c.c. \tag{3.4}$$

This balancing condition can also be obtained [5], [23] from a superpotential of the form $W = \int_X (H + \frac{i}{2}dJ) \wedge \Omega$. In this framework the gaugino condensate can be included via its effective superpotential as well [24] (cf. also [25]). Unfortunately, the moduli space of non-Kähler spaces is still not appropriately understood. For the issue of the stabilization of the complex structure moduli z_i, Kähler modulus T and dilaton S, it would therefore be favorable to have separate control over the $H^{2,1}$ and $H^{3,0}$ sectors. One starts from an effective superpotential description of the H-flux and the gaugino condensate in four-dimensional moduli fields, but it is not quite clear what in the non-Kähler situation the 'complex structure moduli' and 'Kähler moduli' really are. It is one of the goals of the present work to show under which conditions we can consistently set $H^{2,1}$ supersymmetrically to zero and still stabilize all z_i, T, S moduli in a reliable way, while keeping a non-trivial $H^{3,0}$.

One quickly faces a problem since supersymmetry seems to require a non-trivial $H^{2,1}$ once a non-vanishing $H^{3,0}$ is induced through gaugino condensation. The situation is the following: when one is arguing via the potential obtained from dimensional reduction, one finds

$$\langle\operatorname{tr}\lambda\lambda\rangle \neq 0 \implies H^{3,0} \neq 0 \qquad \text{while} \qquad H^{2,1} = 0 = dJ \tag{3.5}$$

However in [3] it was shown, working to lowest order in α', that, in marked contrast to this result, *in an effective four-dimensional supergravity approach*, incorporating H-flux and gaugino condensation via the combined superpotential

$$W = W_H[z_i] + W_{GC}[S] \tag{3.6}$$

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all z_i moduli and the dilaton S could be fixed, but it was not possible to turn on exclusively $H^{3,0}$ flux in a supersymmetric way while having $H^{2,1} = 0$

$$\langle \mathrm{tr}\lambda\lambda \rangle \neq 0 \implies H^{3,0} \neq 0 \implies H^{2,1} \neq 0 \neq dJ \ . \tag{3.7}$$

The reason for this failure is recalled in subsect. 3.4 where we also connect to the work of [13], pointing to inclusion of the periods besides the flux. Let us note that the vacuum found for the combined superpotential W is non-supersymmetric because one still has to take into account the T sector with $D_T W = K_T W \neq 0$. Since $\partial_T W = 0$ one finds that T remains undetermined. In this no-scale model the vacuum energy is zero.

3.2 The Potential From Dimensional Reduction

Starting with a ten-dimensional metric (we follow conventions of [13] which normalizes the Calabi-Yau metric g_{mn}^{CY} to have volume $4\alpha'^3$)

$$ds_{10}^2 = e^{-6\sigma} ds_4^2 + e^{2\sigma} g_{mn}^{CY} dy^m dy^n \ , \tag{3.8}$$

the dimensional reduction of the ten-dimensional action leads to the complete square in the potential

$$V \sim \int_X d^6 y \sqrt{-g^{CY}} \Big(H - \frac{e^{12\sigma}}{16} l_{str}^2 \mathcal{T} \Big)^2 \ . \tag{3.9}$$

Here one has the decompositions (H being closed and harmonic for the standard embedding of unbroken hidden E_8 with $C_G = 30$, and in general up to α' corrections)

$$\mathcal{T} = 2U\Omega + c.c. \tag{3.10}$$

$$H = \alpha\Omega + b^i \chi_i + c.c. \ , \quad i = 1, \ldots, h^{2,1}(X) \ , \tag{3.11}$$

where the gaugino condensate is described by the effective field

$$U = \langle \mathrm{tr}\lambda\lambda \rangle = 16\pi^2 m_{KK}^3 e^{-\frac{2\pi f_{hid}}{C_G}} = e^{-12\sigma} \mu^3 e^{-\frac{2\pi f_{hid}}{C_G}} \tag{3.12}$$

with $\mathrm{Re} f_{hid} = 4\pi/g_{hid}^2$, the Kaluza-Klein scale given by $m_{KK}^3 = e^{-12\sigma} \frac{c}{2} m_{str}^3$, c being an $\mathcal{O}(1)$ numerical constant [13]) and

$$\mu^3 = 8\pi^2 c \, m_{str}^3 \ . \tag{3.13}$$

Furthermore one can define suitably contracted expressions

$$b_i = G_{i\bar{k}} \bar{b}^{\bar{k}} \ , \quad b_{\bar{j}} = G_{l\bar{j}} b^l \tag{3.14}$$

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with the complex structure moduli space metric

$$G_{i\bar{j}} = -\frac{\int \chi_i \wedge \bar{\chi}_{\bar{j}}}{vol/i} , \quad vol/i = \int_X \Omega \wedge \bar{\Omega} .$$ (3.15)

To relate expressions involving fluxes to periods over specific cycles we choose a symplectic basis (A^p, B_q) of $H_3(X, \mathbf{Z})$ for which $A^p \cap A^q = B_p \cap B_q = 0$ and $A^p \cap B_q = \delta_q^p$ with $p, q = 0, \ldots, h^{2,1}(X)$. One then defines the periods

$$X^p = \int_{A^p} \Omega , \quad F_q = \int_{B_q} \Omega$$ (3.16)

and has the 3-cycle $C(H) = e_p A^p - m^q B_q$ dual to H. This means the integrally quantized [2] fluxes are given by (our normalization differs by a factor of π from [2]

$$\frac{1}{2\pi^2 l_{str}^2} \int_{A^p} H = m^p \quad , \quad \frac{1}{2\pi^2 l_{str}^2} \int_{B_q} H = e_q$$ (3.17)

so that one finds

$$h = \frac{1}{4\pi^2 l_{str}^5} \int_X H \wedge \Omega = -\frac{1}{2 l_{str}^3} (e_p X^p - m^q F_q)$$ (3.18)

$$= \frac{1}{4\pi^2 l_{str}^5} \bar{\alpha} \frac{vol}{i}$$ (3.19)

which represents roughly the number of flux quanta [13].

3.3 Balancing Flux and Gaugino Condensation

Let us first assume that H is supported on a 3-cycle C_k with integer flux n_k

$$\frac{1}{2\pi^2 l_{str}^2} \int_{C_k} H_B = n_k$$ (3.20)

and period

$$\Pi_k = \int_{C_k} \Omega .$$ (3.21)

The gaugino condensate then contributes to the potential obtained via dimensional reduction through (where $S = s + i\sigma_S = f_{hid}$)

$$\int_{C_k} \frac{e^{12\sigma}}{16} l_{str}^2 (2\langle \mathrm{tr}\lambda\lambda \rangle \Omega + c.c.) = \frac{c\pi^2}{l_{str}} e^{-\frac{2\pi s}{C_G}} \left(e^{-\frac{2\pi i \sigma_S}{C_G}} \Pi_k + c.c. \right) .$$ (3.22)

Hence minimizing the complete square of the potential gives the following balancing equation between flux and gaugino condensate

$$\boxed{n_k = \frac{c}{2} e^{-\frac{2\pi s}{C_G}} \left(e^{-\frac{2\pi i \sigma_S}{C_G}} \Pi_k + c.c. \right) l_{str}^{-3}}$$ (3.23)

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which has the usual problem of balancing an exponentially small right-hand side (at weak coupling) with an integer (here $(e^{-\frac{2\pi i \sigma_S}{C_G}}\Pi_k + c.c.)l_{str}^{-3} \approx 1$ was assumed in [13]).

One possible way out is to use a possible non-integrality of n_k arising from the Yang-Mills Chern-Simons term [13] (including α' corrections; H becomes non-closed). The Lorentz Chern-Simons term was ignored in [13] but still the full $H = dB + CS_L - CS_{YM}$ has to be solved (potentially taking for A_{YM} just the spin-connection ('spin in the gauge') but deformed by a flat A on a sLag-cycle). Alternatively one may discuss, as we will do next in the effective supergravity approach, the period and its potential ability to compensate the exponential suppression, cf. remarks after (3.36) and before (3.41).[9]

3.4   The Effective Supergravity Approach

Let us now come to the effective four-dimensional description for which we define from the metric (3.8) and the ten-dimensional dilaton ϕ the real moduli

$$s = e^{-(\phi/2-6\sigma)} , \quad t = e^{\phi/2+2\sigma} . \tag{3.24}$$

Together with the corresponding axions they build the complex scalars

$$S = s + i\sigma_S \quad , \quad T = t + i\sigma_T \tag{3.25}$$

and satisfy $e^{12\sigma} = s^{3/2}t^{3/2}$. The complex structure moduli z_i are defined via the periods

$$z_i = \frac{X^i}{X^0} , \quad i = 1,\ldots,h^{2,1}(X) . \tag{3.26}$$

The effective description describes the flux and gaugino condensate effects through the superpotentials (actually a further factor e^{-1} occurs in the normalization of W_{GC} in [3]; this will cause no difference in our argument)

$$W = W_H + W_{GC} \tag{3.27}$$

$$W_H = \frac{4}{\alpha'^4}\int H \wedge \Omega = \frac{4}{\alpha'^4}\bar{\alpha}\frac{vol}{i} = \mu^3\frac{2}{c}h \tag{3.28}$$

$$W_{GC} = -C_G\mu^3 e^{-\frac{2\pi f_{hid}}{C_G}} = -C_G\mu^3 e^{-\frac{2\pi S}{C_G}} . \tag{3.29}$$

Here $\mathrm{Re}f_{hid} = 4\pi/g_{hid}^2 = s$ is the classical result for the gauge kinetic function in the weakly coupled heterotic string. Furthermore one has for the Kähler potential

$$K = -\log 2s - 3\log 2t - \log vol . \tag{3.30}$$

---

[9]One might also use the strongly coupled heterotic string, with the hidden boundary stabilised near the singularity [29], [30], ameliorating the balancing $n \sim e^{-(S-\gamma T)/C_G}$ by having $S - \gamma T \approx 0$.

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When searching for supersymmetric vacua one has to consider the minimization of the corresponding scalar potential V_{Sugra}. Up to a multiplicative Kähler factor, which is of no concern to us here, the effective supergravity potential is ($D_i \equiv D_{z_i}$)

$$V_{Sugra} \sim K^{S\bar{S}} D_S W D_{\bar{S}} \bar{W} + G^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} \tag{3.31}$$

It will be interesting to note that the effective potential, derived by dimensional reduction of the action (3.9),

$$V \sim \left| \alpha + \frac{l_{str}^2}{8C_G} W_{GC} \right|^2 + G_{i\bar{j}} b^i b^{\bar{j}} . \tag{3.32}$$

differs from the four-dimensional supergravity result (cf. [3]).

Vacua with unbroken supersymmetry follow from setting $D_S W = D_T W = D_i W = 0$. However, since our superpotential is T independent, the model in the present case is of no-scale type, which is the reason why the negative $-3|W|^2$ term cancelled. This means that supersymmetry will be broken with non-negative vacuum energy due to a non-vanishing F-term $D_T W = K_T W = -\frac{3}{2t} W \neq 0$. As, without having included an explicit T-dependence, the condition $D_T W = 0$ can not be solved by a finite T we will therefore only demand that $D_S W = D_i W = 0$. The covariant derivatives are given explicitly as follows

$$
\begin{aligned}
D_S W &= -\frac{2\pi}{C_G} W_{GC} + K_S W_{GC} + K_S W_H \\
&= K_S \left[\left(\frac{4\pi}{C_G} s + 1 \right) W_{GC} + W_H \right] \\
D_i W &= D_i W_H + K_i W_{GC}
\end{aligned}
\tag{3.33}
$$

giving finally, by using that $D_i \Omega = \chi_i$,

$$
\boxed{
\begin{aligned}
D_S W &= K_S \left(\left(\frac{4\pi}{C_G} s + 1 \right) W_{GC} + \frac{4}{\alpha'^4} \bar{\alpha} \frac{vol}{i} \right) \\
D_i W &= \frac{4}{\alpha'^4} b_i \frac{vol}{i} + K_i W_{GC}
\end{aligned}
}
\tag{3.34}
$$

Let us first focus on the dilaton equation $D_S W = 0$. It gives the balancing between the flux-period product and the gaugino condensate

$$
-\frac{8\pi^2}{\alpha'^3} (e_p X^p - m^q F_q) = \frac{4}{\alpha'^4} \bar{\alpha} \frac{vol}{i} = -\left(1 + \frac{4\pi}{C_G} s\right) W_{GC}
$$

$$
= \mu^3 (C_G + 4\pi s) e^{-\frac{2\pi S}{C_G}} . \tag{3.35}
$$

Hence, as compared to (3.23), the effective supergravity approach leads to a (well-known) additional factor in the flux condensate balancing equation

$$
\boxed{ h = -\frac{1}{2l_{str}^3} (e_p X^p - m^q F_q) = \frac{c}{2} (C_G + 4\pi s) e^{-\frac{2\pi i \sigma_S}{C_G}} e^{-\frac{2\pi s}{C_G}} }
\tag{3.36}
$$

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Remark: To recover (3.23) one may evaluate $\int_X H \wedge \Omega$ under the assumption that, schematically, H is supported just on C_k (of associated dual cycle D_k in a symplectic basis). Compared with (3.23) the inverse of the dual period appears because one has

$$h = \frac{1}{4\pi^2 l_{str}^5} \int_X H \wedge \Omega = n_k \cdot \frac{1}{2l_{str}^3} \int_{D_k} \Omega \,, \tag{3.37}$$

i.e. one gets in total

$$n_k = \frac{c}{2} \left(C_G + 4\pi s \right) e^{-\frac{2\pi s}{C_G}} \left(\frac{1}{2l_{str}^3} \int_{D_k} \Omega \right)^{-1} . \tag{3.38}$$

Using, however, the full expansion (3.18) instead of (3.37) one gets a matching from (3.36) (up to the $C_G + 4\pi s$ factor). To address the fact that in the weakly coupled regime the right-hand side of (3.36) is much smaller than one, one needs to check the size of the periods appearing on the left-hand side. These are linked to the size of the complex structure moduli such that the questions of stabilization of the complex structure moduli and s have to be treated together.

3.5 Emergence of the $H^{2,1}$ Component

Let us next come to the complex structure equation $D_i W = 0$, from which one gets

$$\frac{4}{\alpha'^4} b_i \frac{vol}{i} = -K_i W_{GC} . \tag{3.39}$$

Together with (3.35) this implies the following fixing of the z_i

$$\boxed{b_i \left(1 + \frac{4\pi}{C_G} s \right) = \bar{\alpha} K_i} \tag{3.40}$$

Hence one arrives at the conclusion [3] that $b_i \neq 0$ and therefore $H^{2,1} \neq 0$, as otherwise α and therefore $H^{3,0}$ would have to vanish or $s \to \infty$.

Note that in the given description of complex structure moduli stabilization it is assumed that (as is has yet to be determined what the consistent complex structure of the underlying Calabi-Yau space X actually is) the H is given as an input as a real three-form; then one has to rotate within the possible complex structures of X until one finds $H^{2,1} = 0$. From this one obtains a fixing of the z_i by using one of the following two reasonings: either as demanding $b_i = 0$ poses $n = h^{2,1}$ conditions on the z_i (this parallels the similar type IIB argument) or exploiting [2] the ensuing proportionality between H and $\Omega + c.c.$, and the integrality of $H = dB$ (if no CS-terms become manifest). Having fixed already the z_i there is no room to satisfy the further conditions $K_i \bar{\alpha}/(1 + \frac{4\pi}{C_G} s) = 0$ on the $K_i(z_j)$, and so on the z_j, what leads to the contradiction (cf. footnote 6 for the IIB case).

So in this framework one finds the interesting feature that it is not possible to turn on in a supersymmetric way (this concerns the z_i and S sectors) an $H^{3,0}$ flux while keeping $H^{2,1} = 0$. This is in marked contrast to the conclusion which would be reached arguing just from the potential coming from dimensional reduction, the difference coming from the $K_i W_{GC}$ term in the second term in (3.31) (the different $(\frac{4\pi}{C_G} s + 1)$ factor coming from the covariant derivative D_S is for this question not essential).

3.6 Implementing a KKLT-Like Two-Step Moduli Stabilization Procedure

In analogy to the type IIB case one can now contemplate the possibility of using the two-step procedure integrate out the z_i in the problem first, i.e. from just $D_i W_H = \int H \wedge \chi_i = \bar{b}_i = 0$ such that H is $(3,0) + c.c.$ and the z_i are fixed, following [2], from the ensuing integrality of (a multiple of) $\Omega + c.c.$. So by implementing the KKLT-like two-step procedure of moduli stabilization, assuming the z_i are heavy enough[10] to be integrated out first, the emergence of $H^{2,1} + c.c.$ could be avoided.

The problem of stabilization at weak coupling and the heterotic discretuum

Finally let us comment on the question of stabilisation between the flux/period-product and the exponentially suppressed gaugino condensate expression in (3.36). In type IIB one needs analogously a hierarchically small value of $W_0 = W_G$ to have T fixed at large volume; similarly here a not small but $\emptyset(1)$ flux value n_k in (3.23) prohibits a balancing with a large S, i.e. being at weak coupling. One possibility is to invoke the fractional flux argument [13] where H is no longer closed as one will not be in the case of the standard embedding where the Chern-Simons terms would cancel. For other proposals cf. for example [27], [28]. In the spirit of the investigations about type IIB string theory related to the discretuum one would argue for a sufficiently small value of W_H (exponentially suppressed for a weak coupling solution for s) just from a heterotic discretuum [3] for large $h^{2,1}$ like in type IIB. But note that heterotically there are only half as many fluxes per modulus, making tunability nearly impossible.[11] We however expect that the completion of H by dJ makes for a fuller analogy with type IIB even in this respect.

Note that a reasoning for a near conifold vacuum as in type IIB [12], [17], to make even at least one period in W_H sufficiently small, meets the following obstacle caused by

---

[10]If the mass of the z_i lies at or above the threshold given by the inverse size of the Calabi-Yau space, then their stabilization has to be discussed within the ten-dimensional theory. In this case one also sees from (3.32) that $b^i = 0$ fixes the z_i.

[11]We thank the referee for bringing this point up.

the reality of H: with the vanishing cycle $A = \mathbf{S^3}$ and dual cycle B

$$\int_A H = M \quad , \quad \int_B H = K \tag{3.41}$$

$$\int_A \Omega = z \quad , \quad \int_B \Omega = \mathcal{G}(z) = \frac{1}{2\pi i} z \ln z + \text{holo.} \tag{3.42}$$

(with integers M and K) one has for the flux superpotential

$$W_H = \int H \wedge \Omega = M\mathcal{G} - Kz \tag{3.43}$$

Then one finds for $K \gg 1$ from the condition for the complex structure modulus z

$$0 = D_z W_H = M\mathcal{G}' - K + K_z W_H \approx \frac{M}{2\pi i} \ln z - K + \mathcal{O}(1) \tag{3.44}$$

(imposing the $D_z W = 0$ condition just for the W_H sector assumes the two-step procedure).

Therefore $z \approx e^{2\pi i K/M}$ rather than being exponentially small. (For a supersymmetric BPS cycle A (in type IIB) the sLag condition shows $\int_A \Omega$ is just a real volume.) So the reality of H prohibits a relation $\int_B H = iK$, needed to stabilize a near-conifold vacuum, which was possible in type IIB where $\int_B G = iK/g_s$ (for a purely imaginary dilaton).

In [5], [24], [23] a complex version of H was used

$$\mathcal{H} = H + \frac{i}{2} dJ \tag{3.45}$$

and led to satisfying results for the corresponding superpotential when compared with the potential coming from dimensional reduction. By contrast in the present paper we put $dJ = 0$ and use the effective four-dimensional supergravity approach where the appropriate moduli space in the case $dJ \neq 0$ is not yet well understood. Using the imaginary component of $\mathcal{H}$ would one bring precisely back to that problem of non-Kählerness. Nevertheless by invoking this possibility one would make an even closer analogy to the type IIB case as one gets a size-dependent imaginary part of $\mathcal{H}$ and thus the analogue (under the substitution $\tau \to T_{het}$ and $T_B \to S_{het}$ which relates heterotic case and type IIB, cf. [10]) to the type IIB three-form

$$G = F - \tau H \tag{3.46}$$

4 Inclusion of a Non-Perturbative Size-Fixing Superpotential

Above the superpotential did not yet depend on the T modulus, i.e. it was of no-scale type and T remained unfixed. Although a volume-modulus stabilizing effect of a non-

trivial $H^{2,1}$ together with dJ is known [4], [5], [6] it is the latter component which we wish to avoid here for the mentioned reason of having a clear-cut notion of Kähler- and complex structure moduli. In the similar framework of the strongly coupled heterotic string the inclusion of a non-perturbative size-fixing superpotential led to a stabilization of the T modulus [29], [30]. There the T modulus was stabilized by balancing the effects of a non-perturbative size-fixing superpotential (induced from open membrane instantons) e^{-T} with the contribution e^{-f_{hid}/C_G} coming from the gaugino condensate (a flux superpotential $\int_X H \wedge \Omega$ can there also be included). Similarly here we will discuss how the inclusion of world-sheet instantons[12] $W_{WSI} = B\, e^{-T}$ changes the above results (cf. also [31]). For its effectivity we assume non-standard embedding[13] (otherwise (1.3)-terms sum to zero). We will first discuss whether it is possible to have $H^{2,1} = 0$ without using the two-step procedure mentioned above.

The complete square structure is now broken by $\partial_T W \neq 0$: the $K^{T\bar{T}} D_T W D_{\bar{T}} \bar{W}$ term of V_{sugra} will not just cancel the $-3|W|^2$ term. Nevertheless stationary points of $\partial V = 0$ can, for supersymmetric vacua, still be found from just the $DW = 0$ conditions.

To demonstrate the unavoidability of the new Hodge type $H^{2,1}$ we ask whether one can find now susy vacua with $b_i = 0$. For this we start from the following superpotential (assuming $h^{1,1} = 1$)

$$
\begin{aligned}
W &= W_H + W_{WSI} + W_{GC} \\
&= \int H \wedge \Omega + Be^{-bT} + Ce^{-aS}
\end{aligned}
\tag{4.1}
$$

One gets (with the notation $W^{S,T} = W_{WSI} + W_{GC}$)

$$
D_i W = b_i \frac{vol}{i} + \partial_i B\, e^{-bT} + K_i W^{S,T}
\tag{4.2}
$$

$$
D_S W = -aCe^{-aS} - \frac{1}{S + \bar{S}} W = -\frac{1}{2s}\left(2as \cdot Ce^{-aS} + W_H + W^{S,T} \right)
\tag{4.3}
$$

$$
D_T W = -bBe^{-bT} - \frac{3}{T + \bar{T}} W = -\frac{3}{2t}\left(2\frac{bt}{3} \cdot Be^{-bT} + W_H + W^{S,T} \right)
\tag{4.4}
$$

Let us at first in (4.2) neglect the contribution $\partial_i B \neq 0$ (this will be remedied below). Then imposing in addition the condition $b_i = 0$ gives $W^{S,T} = 0$, i.e.

$$
Ce^{-aS} = -Be^{-bT}
\tag{4.5}
$$

---

[12] Actually only the product Be^{-iT_I}, and not these factors individually, is strictly well-defined [7].

[13] If one then wants, having included an effect non-perturbative in α', to take into account perturbative α' corrections so that $dH \neq 0$ one replaces (3.11) by $H = H^{3,0} + H^{2,1} + c.c.$ and *defines* $\bar{\alpha}\, vol/i := \int H \wedge \Omega$, $b_i vol/i := \int H \wedge \bar{\chi}_i$ (after having chosen representatives) so that demanding $H^{2,1} = 0$ still implies $b_i = 0$.

So here the complex structure moduli are fixed by $b_i = 0$. Independently of the explicit value $W_0 = \bar{\alpha}\, vol/i$ of W_H (4.3) and (4.4) give already a relation between S and T

$$as \cdot Ce^{-aS} = \frac{bt}{3} \cdot Be^{-bT} \tag{4.6}$$

(4.5) and (4.6) determine[14] s and t but outside the regime of physical validity as the ensuing relation $as = -bt/3$ between the $a, b, s, t > 0$ shows; so one can not have $b_i = 0$.

With $\partial_i B$ included one has from the demand $b_i = 0$ now instead of $W^{S,T} = 0$ that

$$b_i = 0 \quad \Longrightarrow \quad W^{S,T} = -\frac{\partial_i B}{K_i} e^{-bT} \tag{4.7}$$

i.e. one finds instead of (4.5) now

$$Ce^{-aS} = -\left(\frac{\partial_i B}{B\, K_i} + 1\right) Be^{-bT} \tag{4.8}$$

Note that the interpretation would now be different than before. In the previous section we considered the H-flux as a given real form and a fixing of the b_i as in (3.40) was considered as fixing the complex structure moduli z_i. Now we would have to regard H as adjustable, impose the demand $b_i = 0$ and get as conditions for the z_i the relations (4.8) where one gets for the z_i (still entangled there with the S and T yet to be determined) the conditions (for all $i = 1, \ldots, h^{2,1}$; the constant k defined on the solution pair (S, T))

$$\frac{\partial_i B}{B K_i} + 1 = k := -Ce^{-aS}/Be^{-bT} . \tag{4.9}$$

So essentially this condition on the $\partial_i B/(BK_i)$ would determine the z_i (i.e. modulo the coupled determination of the S and T).[15] Now (4.6) gives, with $a, b > 0$, the condition $Ce^{-aS}/(Be^{-bT}) \in \mathbf{R}^{>0}$, i.e. $k \in \mathbf{R}^{<0}$; the point here is that previously, without $\partial_i B$, the parameter k was just 1 leading to a contradiction to $k \in \mathbf{R}^{<0}$. So one would now have consistently $b_i = 0$ in the sense that for a given α the S, T and the z_i are determined with having also $b_i = 0$. (4.3) and (4.4) show that, as $W_H \neq 0$, there exists a second solution of finite s and t besides the runaway solution $s, t \to \infty$.

So the inclusion of $W_{WSI} = Be^{-bT}$ would make it possible, taking into account that $\partial_{z_i} B \neq 0$, to solve consistently with $H^{2,1} = 0$ *if a $H^{3,0} \neq 0$ could be suitably adjusted.*

---

[14]The earlier case is included formally as the degenerate case $B = 0$ not corresponding to a finite s.

[15]In principle one could have tried to use this reasoning to solve (3.40) with the conditions $K_i = 0$ for the z_j and a similar adjustment of H as here; but without W_{WSI} one would not have fixed the T modulus as it does otherwise the $H^{2,1}$.

4.1 Remarks on the Problem of Adjusting the H-Field

When including world-sheet instanton effects one has to work outside the standard embedding as otherwise contributions coming from the curves in the same cohomology class would sum up to zero. So (with $\omega_{YM} = \text{tr}(AdA + \frac{2}{3}A^3)$ being the Chern-Simons form)

$$H = dB + CS \quad \text{where} \quad CS = \frac{\alpha'}{4}(\omega_L - \omega_{YM}) \implies dH = \frac{\alpha'}{4}\text{tr}(R \wedge R - F \wedge F) \quad (4.10)$$

With $F \neq R$ now H will also no longer be just dB and so[16] $dH \neq 0$ generically and H can not be decomposed as a sum (3.11) of closed forms. As relevant for the $DW = 0$ conditions were only the integrals $W_H = \int H \wedge \Omega$ and $\int H \wedge \bar\chi_i = 0$ we just call, even without a decomposition (3.11), the values of these integrals $\bar\alpha$ and b_i (cf. footn. 13).

The change of interpretation mentioned after (4.8) deserves more discussion. Usually, when fixing the z_i by a flux, H is given first as a real form on the underlying real manifold and then the complex structure is fixed as conditions emerge from $D_iW = 0$. For example, if H would be built from working in the standard embedding $F = R$ such that $H = dB$ and so H is closed, a condition like $b_i = \int H \wedge \bar\chi_i = 0$ would amount to an equation $H = \alpha\Omega + c.c.$ such that $\alpha\Omega$ has to have integral periods what fixes the z_i [2].

However above a different reasoning was tried: the z_i were fixed from the remaining (after putting $b_i = 0$ in $D_iW = 0$) condition (4.8) and an H of $b_i = 0$ was treated as if adjustable independently. So we have to ask whether it is possible to turn on[17] an $H = H^{3,0} + c.c. \neq 0$ *without posing thereby additional conditions on the z_i besides* (4.8).

Actually, just when $H = H^{3,0} + c.c. = (dB)^{3,0} + (CS)^{3,0} + c.c.$, one finds that H is already closed[18], i.e. $H = \alpha\Omega + c.c.$. But some $dB^{2,1}$ and $CS^{2,1}$ may cancel here. Note that one can not split usually the contributions of dB and CS as even the neutral field B gauge transforms to achieve invariance of H under gauge transformations

$$\delta A = d\Lambda + [A, \Lambda] \longrightarrow \delta\omega_{YM} = d\,\text{tr}\,\Lambda dA \implies \delta B = \frac{\alpha'}{4}\,\text{tr}\,\Lambda dA \quad (4.11)$$

One needs a non-trivial A as $dB = \alpha_{dB}\Omega + c.c.$ would have to be integral and fix the z_i.

---

[16] understood as a four-form; considering just cohomology the integral of this closed four-form over a four-cycle gives the number of fivebranes wrapping the dual two-cycle necessary for anomaly cancellation

[17] if also $H^{3,0} = 0$ the equations (4.3), (4.4) have no finite solution in (s, t) as is easily seen

[18] as the mentioned Hodge-types for CS give under d the types $(3, 1) + c.c.$ and have therefore to vanish as $dCS = \text{tr}(R \wedge R - F \wedge F)$ has type $(2, 2)$. Note further that $dH = 0$ with $H \neq 0$ implies $\text{tr}F \wedge F = \text{tr}R \wedge R$ with $F \neq R$. To potentially achieve this one may consider F corresponding to deformations V of TX, cf. the deformations Q of $Q_0 = TX \oplus \mathcal{O}$ in [32]. This uses the expansion in α'/t when discussing corrections to the equations of motion and the solvability of (4.10) in cohomology (i.e. assuming $c_2(V) = c_2(X)$, the absence of five-branes) is then sufficient to solve (4.10) for H as form.

On the use of CS-fluxes through special Lagrangian 3-cycles

We discuss whether one can use advantageously fluxes through special Lagrangian 3-cycles, cf. [13]. For this we ignore first the dB- and ω_L-part (possibly balancing the Bianchi identity with additional five-branes) and view the remaining CS-fluxes of Hodge-type $(3,0) + c.c.$ as fluxes through special Lagrangian 3-cycles Q (cf. [13]).

A Lagrangian 3-cycle Q of $J|_Q = 0$ is special Lagrangian (sLag) if also $\operatorname{Im}\Omega$ restricts to zero on it. On such cycles[19] Ω restricts to a multiple of the volume form

$$J|_Q = 0 \quad , \quad \operatorname{Im}\Omega|_Q = 0 \qquad \Longrightarrow \qquad \operatorname{Re}\Omega|_Q = \gamma\, vol_Q \tag{4.12}$$

Now, how can one realize a closed $H = H^{3,0} + c.c. \neq 0$, i.e. $H = \alpha\Omega + c.c.$ (and not purely dB)? For this let C_Q be a real three-form supported on a rigid sLag-cycle Q, i.e. $C_Q = h_q\, vol_Q$ with h_Q real. With $\Omega|_Q = \gamma\, vol_Q$ one finds, as $\frac{1}{\gamma}\Omega|_Q$ is a real form, indeed

$$C_Q = h_q\, \frac{1}{\gamma}\Omega|_Q \qquad \Longrightarrow \qquad \int_Q C_Q \sim \int_Q H = \int_Q \alpha\Omega + c.c. \tag{4.13}$$

(π as in footnote 19) the latter from the Hodge-type of the closed form H.

The sole use of a flat gauge connection A to build H would still be insufficient as one has also for a pure $H = -\frac{\alpha'}{4}\omega_{YM}$ with A flat a (fractional) quantization like in (3.17)

$$\frac{1}{2\pi^2\alpha'}\int_Q H = -\frac{1}{8\pi^2}\int_Q \omega_{YM} \in \frac{1}{p}\mathbf{Z}/\mathbf{Z} \tag{4.14}$$

$((p \in \mathbf{Z})$; the integral being a topological invariant on the moduli space of flat connections). $H \sim \omega_{YM}$ would then fix again the z_i as did before the quantized $H = dB$.

So interpreting the Hodge-type condition $(3,0) + c.c.$ of the sought-after H-flux as a condition of being such a $(\pi^*)C_Q$, i.e. being supported on a sLag 3-cycle Q, one could be able to turn on a non-flat A on Q without imposing thereby forbidden additional conditions on the z_i (which might have originated from the Hodge-type restriction)

$$dB + CS = h_q vol_Q \tag{4.15}$$

But turning on ω_{YM} on Q shows just how a Hodge-type $(3,0) + c.c.$ could occur in general and makes not clear how to satisfy the supersymmetry conditions $F = F^{1,1}$ with $g^{i\bar{j}}F_{i\bar{j}} = 0$ for $F \neq 0$ or how to obtain a consistent package $H = CS = H^{3,0} + c.c.$ (ω_L is ignored in [13]). *Because of this difficulty we assume the KKLT two-step procedure* where terms $K_i W^{S,T}$ and $\partial_i B e^{-bT}$ in $D_i W = 0$ (and the first equation in (4.9)) do not arise.

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[19]Examples are provided by the real points of a Calabi-Yau manifold X with possesses a real structure (an antiholomorphic involution τ with $\tau J = -J$ and $\tau\operatorname{Im}\Omega = -\operatorname{Im}\Omega$; the real points of X are the fixed points of τ). We restrict the attention to *rigid* special Lagrangian 3-cycles Q which are then known to have $b_1(Q) = 0$. Locally near such a cycle the Calabi-Yau geometry of X looks like T^*Q and globally such a Q could occur as the base of a fibration $\pi : X \to Q$ of X by special Lagrangian three-tori.

4.2 One-Loop Corrections

Above it was assumed that the only T dependence comes from W_{WSI}, neglecting one-loop corrections [34] to f_{hid} and effects from warp factors $e^{12\sigma} = s^{3/2}t^{3/2}$. Both of these assumptions would have to be modified when considering the strongly coupled heterotic string.

When H was decomposed in Hodge-types in (3.11) α' corrections were neglected so that H could in particular assumed to be closed; this starting point has to be modified if either one-loop corrections to f_{hid} are included in the weakly coupled heterotic string framework or if the strongly coupled heterotic string is discussed. In the latter case the $f_{hid} = S + \beta T$ with $\beta = \mathcal{O}(1/100)$ of the weakly coupled case is replaced by $S + \gamma T$ with $\gamma = \mathcal{O}(1)$ and this 'correction', which in the strongly coupled case is inevitable, is directly related to the non-trivial part of the Bianchi identity for H_3 resp. G_4, i.e. its non-closedness. Note that a T dependence in f_{hid} which in the weakly coupled heterotic string is included as a one-loop correction is in the strongly coupled heterotic string included already by the variation of the Calabi-Yau volume along the x_{11}-interval [33].

We show now that the inclusion of W_{WSI} will solve a problem which arose in the analysis of [13]. First in a vacuum without five-branes one has from the condition $c_2(V_1) + c_2(V_2) = c_2(TX)$ for the one-loop corrections to the gauge-kinetic functions

$$f_{obs/hid} = S \mp \beta T \tag{4.16}$$

where $\beta = \frac{1}{8\pi^2} \int J \wedge \left(c_2(V_2) - c_2(V_1)\right)$. The combined conditions $D_S W = 0 = D_T W$ in the case without $W_{WSI} = Be^{-bT}$ gave then in [13] the condition

$$3s = \beta t \tag{4.17}$$

This caused a problem: besides the fact that one needs to have $\beta > 0$ so that the observable sector is more strongly coupled than the hidden one, one finds as a more serious consequence a not well-understood strong coupling transition as one gets a negative $\mathrm{Re} f_{obs} = -2s < 0$.

This problem is avoided in our approach as this time (4.17) is replaced by[20]

$$\boxed{3s = \left(\beta - \frac{b}{ak}\right)t} \tag{4.18}$$

(where now actually $k := -W_{GC}/W_{WSI} = -Ce^{-a(S+\beta T)}/Be^{-bT}$). Here we found before, with $\beta = 0$, from (4.18) that $k \in \mathbf{R}^{<0}$. Now we see that one may solve both mentioned

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[20]Note that k is defined by the second equation of (4.9) and the first equation of (4.9) is absent in the two-step procedure employed now.

problems of [13]: we can choose now the more standard choice $\beta < 0$, and have from $\mathrm{Re} f_{obs} = -2s - \frac{b}{ak} t$ now that one is not necessarily led to the strong coupling transition because $\mathrm{Re} f_{obs}$ can now be positive for $k \in \mathbf{R}^{<0}$; more precisely this happens for $2a |k| s < bt$ (demanding also the stronger $\mathrm{Re} f_{hid} > 0$ gives $b/a|k| > 4|\beta|$).

To corroborate these claims, let us start from the full superpotential

$$W = W_H + W_{WSI} + W_{GC} \tag{4.19}$$

$$= \int H \wedge \Omega + Be^{-bT} + Ce^{-a(S+\beta T)} , \tag{4.20}$$

which includes the one-loop correction to the gaugino condensation superpotential. Our goal is to solve

$$D_S W = K_S \Big(2as\, W_{GC} + W \Big) = 0 \tag{4.21}$$

$$D_T W = K_T \Big(2\frac{bt}{3} W_{WSI} + 2\frac{a\beta t}{3} W_{GC} + W \Big) = 0 . \tag{4.22}$$

To this end, let us subtract the first from the second equation with the result

$$\frac{bt}{3} W_{WSI} = a\Big(s - \frac{\beta t}{3} \Big) W_{GC} , \tag{4.23}$$

which is nothing but (4.18). To stay within the weakly coupled heterotic string regime, we have to impose $s \gg |\beta t|$ and check its consistency later at the critical point. Both prefactors are therefore positive. Moreover, to trust the effective supergravity analysis, both s and t should be considerably larger than one. We can then approximate both sides of the equation just through the superpotentials alone (the prefactors written as exponentials contribute only logarithmically instead of linearly to the exponent and can hence be neglected at sufficiently large s and t). This leads[21] to $W_{WSI} \simeq W_{GC}$, hence fixing $k \simeq -1$, and gives us, by a similar reasoning as before suppressing the prefactors in front of the exponentials, the relation

$$\Big(\frac{b}{a} - \beta \Big) T \simeq S . \tag{4.24}$$

We can now adopt (4.21) as the second equation determining S and T besides (4.23). Within the same approximation as before it leads to $W_{GC} + W_H \simeq 0$ which is essentially

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[21]The balancing of gaugino condensation with open membrane instantons is known from the strongly coupled heterotic string to lead to a stabilization of the orbifold-length (dilaton) [30]. Indeed for an unbroken hidden E_8 (as opposed to a hidden gauge group of much smaller dual Coxeter number) the orbifold-length becomes thus stabilized at rather small values close to weak coupling. Here the open membrane instantons become heterotic world-sheet instantons and we arrive at the balancing between W_{WSI} and W_{GC}.

identical to (3.36) and can be solved with large s either by having fractional flux [13] or by making use of the heterotic discretuum [3], probably allowing for non-Kählerness of the background, as discussed in section 3.

Two comments are now in order. First, for a hidden E_8 gauge group we find $a = 2\pi/C_G = 2\pi/30$ whereas $b = 1$. Thus, adopting a value for β of $\varnothing(1/100)$, as appropriate for the weakly coupled heterotic string, we obtain from (4.24) that $5T \simeq S$ at the critical point. Hence, at the critical point the one-loop correction to $f_{obs/hid}$ becomes $\beta T = \varnothing(S/500)$ which is indeed much smaller than the tree level result S, showing that the critical point is consistently located in the weakly coupled regime. Notice that this is not the case when one neglects the world-sheet instanton superpotential as then the critical point becomes characterized by (4.17) which implies a one-loop "correction" βt which is three times as large as the tree level result s. The critical point in this latter case is therefore situated outside the weakly coupled string regime. Second and closely related to this first comment about the smallness of the one-loop correction in the case with world-sheet instantons, we do obtain a positive $\mathrm{Re}f_{obs} > 0$ at the critical point where

$$\mathrm{Re}f_{obs} = s - \beta t = \left(\frac{\frac{b}{a} - 2\beta}{\frac{b}{a} - \beta}\right) s \, . \tag{4.25}$$

Since $\beta = \varnothing(1/100)$ is much smaller than $b/a \simeq 5$ the value is clearly positive. This is again in contrast to the case without world-sheet instanton contribution for which $\mathrm{Re}f_{obs} = -2s$ came out to be negative.

Hence the inclusion of $W_{WSI} = Be^{-bT}$ makes it also possible (besides T stabilization without $H^{2,1}$) to avoid the occurrence of the uncontrolled transition found in [13] while keeping the 1-loop correction small.

Stability of the vacuum

Finally we check stability of these vacua (for parameter values where the axions σ, τ at the solution are zero) via the criterion of [10]. With $W_{eff}^T[T] = W_H + Be^{-bT}$ from (1.7) the stability parameter $\eta = tW_{eff}^{T\,\prime\prime}/W_{eff}^{T\,\prime}$ becomes $\eta = -bt$ and the stability criterion

$$|\eta - 1| = |bt + 1| > 1 \tag{4.26}$$

(in leading order in $1/(as)$) is with $b > 0$ automatically fulfilled. Actually and more appropriately here, because of the relation $\eta = 3ask$ following from (4.6) and (4.9), the reasoning comparing orders in $1/(as)$ has to be slightly reconsidered; the conclusion of stability, in the sector $as \gg 1$ relevant for us, remains unchanged.

The inclusion of the previously mentioned one-loop corrections should not change this result as they have to be small and hence cannot change the sign of V''.

Remark 1: The possibility of flux-less solutions

With the inclusion of the mentioned one-loop effect it becomes possible to solve the $DW = 0$ conditions consistently even with $b_i = 0$, without having to worry about further possible conditions which the latter constraints might impose when just setting H to zero. Previously this was impossible as we recall now, as well.

We will show that $D_S W = 0 = D_T W$ can now have a solution besides the runaway solution $s, t \to \infty$ (the $D_i W = 0$ conditions determine then the z_i). With $W^{S,T} = W_{WSI} + W_{GC}$ one has

$$D_S W^{S,T} = K_S \left(2as\, W_{GC} + W^{S,T} \right) \tag{4.27}$$

$$D_T W^{S,T} = K_T \left(2\frac{bt}{3} W_{WSI} + 2\frac{a\beta t}{3} W_{GC} + W^{S,T} \right) \tag{4.28}$$

with the ensuing condition for a non-runaway solution

$$(2as + 1)\left(2\frac{bt}{3} + 1 \right) = 2\frac{a\beta t}{3} + 1 \tag{4.29}$$

The point here is that this condition, as a, s, b, t are positive, only led to a contradiction in the previous case of $\beta = 0$, but no longer now. The system is now generically be solvable.

Note again that here the somewhat more non-standard choice $\beta > 0$ would have to be made. This implies furthermore that (4.18) now gives[22] $k \in \mathbf{R}^{>0}$ which would bring one back, with $\mathrm{Re} f_{obs} = -2s - \frac{b}{ak}t$, to the transition problem.[23]

Remark 2: One-loop corrections which depend on the complex structure moduli

Actually, not only in the exponent of $W_{GC} = Ce^{-aS}$ a Kähler modulus dependence should be included as a one-loop threshold effect, but also a z_i-dependence of C should be considered (as was already done for B). As described in [35] for the moduli-dependence of the threshold-corrections Δ the Ray-Singer torsion will be relevant. More precisely if the relevant E_8 is broken by a gauge bundle V of structure group H to a (simple) group G with $\mathbf{248} = \oplus_k (R_k, r_k)$ with respect to $G \times H$ one finds (up to an additive piece $\Delta(E_8) = 30 RS(\mathbf{C})$) $\Delta_G = \sum_k C_{R_k} RS(V_{r_k}) = 12 F_1$. Inclusion of this complex structure

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[22]because one has $3s - \beta t < 0$, as one can see from rewriting (4.29) as $2asbt + a(3s - \beta t) + bt = 0$.

[23]The conclusions of this one and the previous subsection will not change if one invokes the correction $-\log\left((T + \bar{T})^3 + E\right)$ with $E > 0$ to the Kähler potential in the T sector.

moduli dependence leads to the replacement of (4.2) by

$$D_i W = b_i \frac{vol}{i} + \frac{\partial_i B}{B} W_{WSI} + \frac{\partial_i C}{C} W_{GC} + K_i (W_{WSI} + W_{GC}) \tag{4.30}$$

which means that, after imposing the demand $b_i = 0$, (4.8) reads now

$$\left(\frac{\partial_i B}{B K_i} + 1 \right) W_{WSI} + \left(\frac{\partial_i C}{C K_i} + 1 \right) W_{GC} = 0 \tag{4.31}$$

Acknowledgements

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